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NEW QUESTION.

36. A number of Discussions have been published in this department relating to cubic and biquadratic equations (cf. Vol. XXXV, p. 29, pp. 268–269 and 343–347; Vol. XXIV, pp. 136–137 and 436–439; Vol. XXIII, pp. 314–315). Below are given a number of questions sent in by Professor Harris Hancock of the University of Cincinnati which relate to the cubic and biquadratic and might, perhaps, more properly be proposed as problems were it not for the advantage to be gained, if possible, by treating them all, or at least several of them, in one discussion.

1. For what values of n can $\cos 2\pi/n$ be expressed in the form $(a + \sqrt{b})/c$ where a, b and c are integers?

2. Write the biquadratic in the form $ax^4 + 4bx^3 + 6cx^2 + 4dx + e = 0$. Show that its reducing cubic may be expressed by means of a determinant of the third order which when expanded is

$$4y^3 - g_2y - g_3 = 0,$$

where $g_2 = ae - 4bd + 3c^2$, and

$$g_3 = \begin{vmatrix} a & b & c \\ b & c & d \\ c & d & e \end{vmatrix} = ace - ad^2 - eb^2 - c^3 + 2bcd.$$

3. For the same biquadratic show that

$$a^3(x_0 + x_1 - x_2 - x_3)(x_0 + x_2 - x_1 - x_3)(x_0 + x_3 - x_1 - x_2) = 32(3abc - a^2d - 2b^3)$$

without making any use of symmetric functions.

4. If x_0, x_1, x_2 are the roots of a cubic and D its discriminant, show that x_1 is a rational function of x_0 and $\sqrt{D}$. Derive a much simpler relation than that given in Serret, *Cours d'Algèbre Supérieure*, 5th ed., No. 511.

5. If x_0, x_1, x_2, x_3 are the roots of a biquadratic, D its discriminant, e_1, e_2, e_3 the roots of the reducing cubic, show that x_1 is a rational function of x_0, e_1, e_2, e_3 and consequently also of x_0, e_1 and $\sqrt{D}$.

6. If the biquadratic

$$a_0x^4 + 4a_1x^3 + 6a_2x^2 + 4a_3x + a_4 = 0 \tag{1}$$

has a double root, show that the reducing cubic (cf., for example, Burnside and Panton's *Theory of Equations*)

$$t^3 + 3Ht^2 + \left(3H - \frac{a^2I}{4}\right)t - G^2/4 = 0 \tag{2}$$

has a root in common with the cubic

$$8t^3 + 12Ht^2 + G^2 = 0; \tag{3}$$

and conversely, if (2) and (3) have a common root, then (1) has a double root.

DISCUSSIONS.

Concerning Haversines in Plane Trigonometry.

By G. W. Evans, Charlestown High School, Boston, Mass.

The work for the S. A. T. C. in American colleges has called attention to the use of haversines. It appears not to be generally known that plane trigonometry can be simplified, much as to teaching and somewhat as to computation, by the use of these ratios.

The haversine is defined as half the versed sine; that is,

$$\text{hav}\, A = \frac{1 - \cos A}{2} = \sin^2 \frac{A}{2}.$$

For the case where a right triangle is given by the hypotenuse c and one side, b, nearly equal to c, there formerly appeared in elementary text-books the formula

$$\sin \tfrac{1}{2}A = \sqrt{\frac{c - b}{2c}}$$